Flipflop stationary state

exa11

Variables and parameters:
y_1, y_2: Voltages in inverter 1 ($y_1 = U_1,\ y_2 = V_1$)
y_3, y_4: Voltages in inverter 2 ($y_3 = U_2,\ y_4 = V_2$)
λ: voltage supply

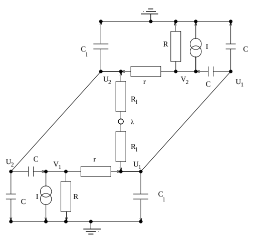
System of equations:

$$U_1\left(\frac{1}{R_l}+\frac{1}{r}\right) - V_1\frac{1}{r} - \lambda\frac{1}{R_l} = 0$$
$$I(U_1,U_2) - U_1\frac{1}{r} + V_1\left(\frac{1}{R}+\frac{1}{r}\right) = 0$$
$$U_2\left(\frac{1}{R_l}+\frac{1}{r}\right) - V_2\frac{1}{r} - \lambda\frac{1}{R_l} = 0$$
$$I(U_2,U_1) - U_2\frac{1}{r} + V_2\left(\frac{1}{R}+\frac{1}{r}\right) = 0$$

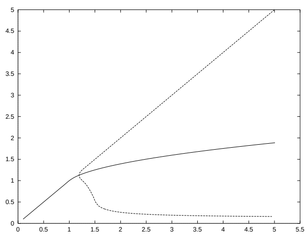
$$I(a,b) := \begin{cases} 0 & \text{for } b \le V_0 \\ \beta(b-V_0)^2(1+\delta a) & \text{for } V_0 < b < V_0 + a \\ \beta a[2(b-V_0)-a](1+\delta a) & \text{elsewhere} \end{cases}$$

$$R_l = 4.5 * 10^5,\ R = 10^{15},\ r = 60$$
$$V_0 = 1,\ \beta = 8.5\cdot 10^{-6},\ \delta = 0.02$$

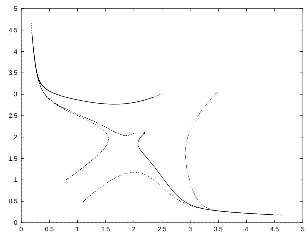
Figures:

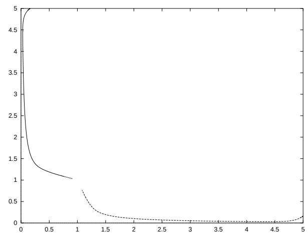
1 

Two identical inverters coupled to a flipflop

2 

Bifurcation diagram y_1 versus λ.

3  (y_1, y_3) phase diagram for $\lambda = 5$.

4  (y_1, y_3) phase diagram for $\lambda = 1$.

Comments and explanations:

Two MOSFET inverters are coupled to a flipflop, see Figure 1. The equations describe the stationary state. Because of the two identical inverters this problem has a natural symmetry, and we may expect bifurcation. In fact there is a bifurcation for $\lambda_0 \approx 1.192$, see Figure 2.

For the stationary states we have $y_1 \approx y_2$ and $y_3 \approx y_4$. For $\lambda = 5$ there are three stationary states with

$y_1 = 5$, $y_3 = 0.16087$ (stable)
$y_1 = y_3 = 1.88577$ (unstable)
$y_1 = 0.16087$, $y_3 = 5$ (stable).

The dynamical behavior for $\lambda = 5$ is indicated in the phase diagram of Figure 3. Some trajectories are shown that approach the stable states.

The model is based on material kindly delivered by Siemens, München.