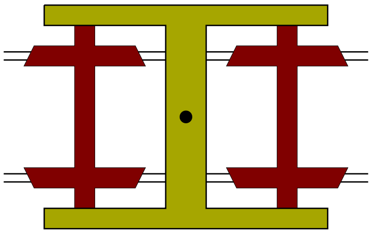
Railway Bogie

exd10

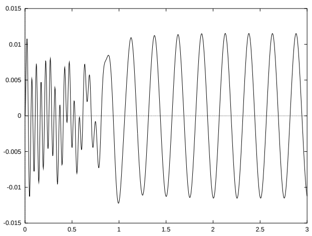
Variables and main parameters:

y_1, y_3, y_5: lateral displacement of front axle, rear axle, bogie frame
y_2, y_4, y_6: yaw displacement of front axle, rear axle, bogie frame
y_7: rolling displacement of bogie frame
v: speed of train (bifurcation parameter)

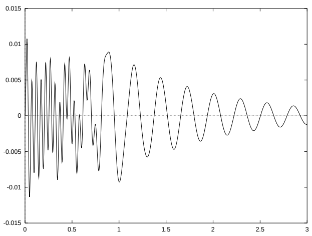
Figures:

1 

A railway bogie with two axles, schematically

2 

attracting periodic orbit for $v = 70$, $y_5(t)$ for $0 \leq t \leq 3$, initial vector $y_1(0) = 0.1$, $y_i(0) = 0$ for $1 \leq i$

3 

transient phase close to the attracting stationary state for $v = 60$, $y_5(t)$ for $0 \leq t \leq 3$, initial vector $y_1(0) = 0.1$, $y_i(0) = 0$ for $1 \leq i$

Comments and explanations:
Original source of the model:

N.K. Cooperrider: The Hunting Behavior of Conventional Railway Trucks. Transactions of the ASME; Journal of Engineering for Industry **94** (1972), 752–762

Further references:

Ch. Kaas-Petersen: Chaos in a Railway Bogie. ACTA Mechanica **61** (1986), 89–107
U. Galvanetto, L. Briseghella, S.R. Bishop: Optimal Axle Distance of a Railway Bogie. Int. J. Bifurcation & Chaos **7** (1997), 721–732

The number of freedoms of this wheelset is 7, and the system consists of 14 first-order differential equations. The stationary solution $y_i \equiv 0$ for all i loses its stability for the parameter value $v_0 \approx 68.66$ mph through a Hopf bifurcation with a hard loss of stability. For a special initial state the transition to the stable periodic orbit is illustrated in Figure

2, whereas in Figure 3 the oscillation is attracted by the stable stationary state. As has been pointed out by Brian Hassard, the Hopf bifurcation is tricky as there are problems with smoothness at the origin. This is explained in Hassard's worksheet, where also the equations and other information is found.