Hodgkin–Huxley Nerve Model

exd14

Variables and parameters:

$U(t)$: membrane potential of a nerve cell

$V_1,\ V_2, V_3$: artificial variables defining permeability of the membrane

I_a: applied current ($\lambda := I_a$ bifurcation parameter)

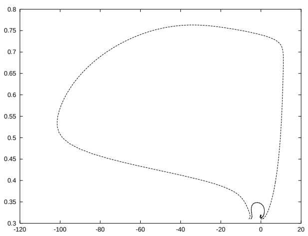
$$
\begin{aligned}
I_a &= \dot{U} + 36V_1^4(U-12) + 120V_2^3V_3(U+115) + 0.3(U+10.613),\\
\dot{V}_i &= \alpha_i(U)(1-V_i) - \beta_i(U)V_i, \quad i = 1,2,3.
\end{aligned}
$$

transfer rates:

$$
\begin{aligned}
\alpha_1 &= 0.01(U+10)/\left(\exp\tfrac{U+10}{10}-1\right), & \beta_1 &= 0.125\exp(U/80),\\
\alpha_2 &= 0.1(U+25)/\left(\exp\tfrac{U+25}{10}-1\right), & \beta_2 &= 4\exp(U/18),\\
\alpha_3 &= 0.07\exp(U/20), & \beta_3 &= 1/\left(\exp\tfrac{U+30}{10}+1\right).
\end{aligned}
$$

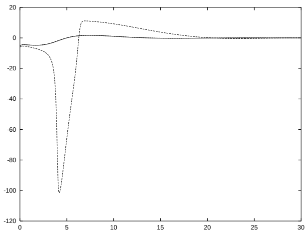
Figures:

1



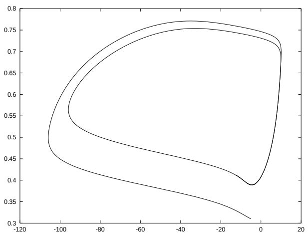
(U, V_1)-phase diagram of the Hodgkin–Huxley equation; two trajectories: $U(0) = -5$ and -6, with $V_1(0) = 0.31$, $V_2(0) = 0.05$, $V_3(0) = 0.6$ and $I_a = 0$.

2



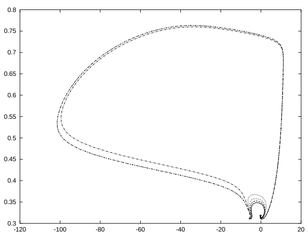
$U(t)$ for $0 \le t \le 30$; Hodgkin–Huxley equation; two trajectories: $U(0) = -5$ and -6, with $V_1(0) = 0.31$, $V_2(0) = 0.05$, $V_3(0) = 0.6$ and $I_a = 0$.

3

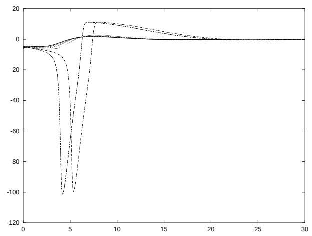


(U, V_1)-phase diagram of the Hodgkin–Huxley equation; trajectory approaching a limit cycle. $U(0) = -5$, $V_1(0) = 0.31$, $V_2(0) = 0.05$, $V_3(0) = 0.6$ and $I_a = -10$.

4



(U, V_1)-phase diagram of the Hodgkin–Huxley equation; six trajectories $U(0) = U_0$, with $V_1(0) = 0.31$, $V_2(0) = 0.05$, $V_3(0) = 0.6$ and $I_a = 0$; $U_0 = -5.0/-5.2/-5.4/-5.6/-5.8/-6.0$

5 

$U(t)$ for $0 \leq t \leq 30$; membrane potential of the Hodgkin–Huxley equation; six trajectories $U(0) = U_0$, with $V_1(0) = 0.31$, $V_2(0) = 0.05$, $V_3(0) = 0.6$ and $I_a = 0$; $U_0 = -5.0/-5.2/-5.4/-5.6/-5.8/-6.0$

Comments and explanations:
Original source of the model: A. L. Hodgkin, A. F. Huxley: A quantitative description of membrane current and its application to conduction and excitation in nerve. J. Physiol. **117** (1952) 500–544.
The experiments reported in Figures 1–5 concentrate on the importance of *threshold values*. Here it is essential whether the initial voltage U_0 is below or above a threshold: the cell will fire for $U_0 = -6$, and will basically be quiet for $U_0 = -5$; compare Figures 1 and 2. Figures 4 and 5 depict in addition the dynamics of some intermediate values.

Figure 3 shows a simulation that approaches a limit cycle. Taking the applied current as bifurcation parameter, $\lambda = I_a$, we find a Hopf bifurcation for $\lambda \approx -9.78$ with an interval of bistability of approximately $-9.78 < \lambda < -5.5$.