Chemical Reaction with Autocatalytic Step

exd6

Variables and parameters:

- $y_i(t)$: concentrations of chemicals
- γ: flow ($\gamma = 3$)
- λ: velocity coefficient of autocatalytic step (bifurcation parameter)

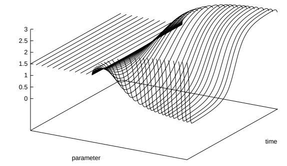
Model equations:

$$\begin{aligned}
\dot{y}_1 &= \gamma - y_1 - \lambda y_1 y_3 \\
\dot{y}_2 &= y_1 - y_2 y_3 \\
\dot{y}_3 &= y_2 y_3 - \lambda y_1 y_3
\end{aligned}$$

Hopf Bifurcation at $\lambda_0 = 1.30176$ with initial period $T_0 = 6.03555$

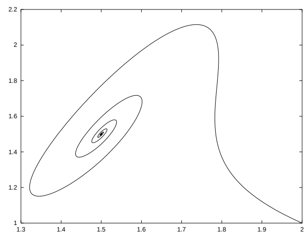
Figures:

1



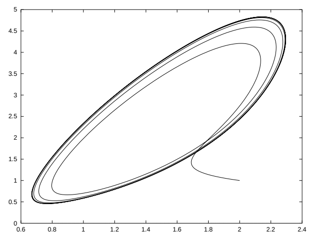
Stable solutions, $y_1(t;\lambda)$ for $0.2 \le \lambda \le 3$; the time is normalized to unity. Stationary and periodic orbits meet in a Hopf bifurcation.

2



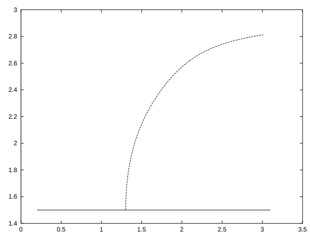
$\lambda = 1$, projection to the (y_1, y_2) plane. Initial values: $y_1 = 2$, $y_2 = y_3 = 1$. Dynamics of a stable focus.

3



$\lambda = 1.5$, projection to the (y_1, y_2) plane. Initial values: $y_1 = 2$, $y_2 = y_3 = 1$. Dynamics of a stable limit cycle.

4



Bifurcation diagram $y_1(0)$ versus λ; Hopf bifurcation at $\lambda_0 = 1.30176$.

Comments and explanations:
Original source of the model: H.-J. Krug, L. Kuhnert: Ein oszillierendes Modellsystem mit autokatalytischem Teilschritt. Z. Phys. Chemie **266** (1985) 65–73.
The flow γ describes the input of y_1; y_1, y_2, y_3 are dimensionless variables. For large values of λ, a relaxation oscillation develops.